



# Structural Equation Modeling

A Quantitative Methods Consulting Center Workshop

Slides and materials available at:

[www.matthewmadison.com/workshop](http://www.matthewmadison.com/workshop)



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# Quantitative Methods Consulting Center (QMC)

- QMC purpose is to provide quantitative research support and training for members of the MFECOE
- QMC Team



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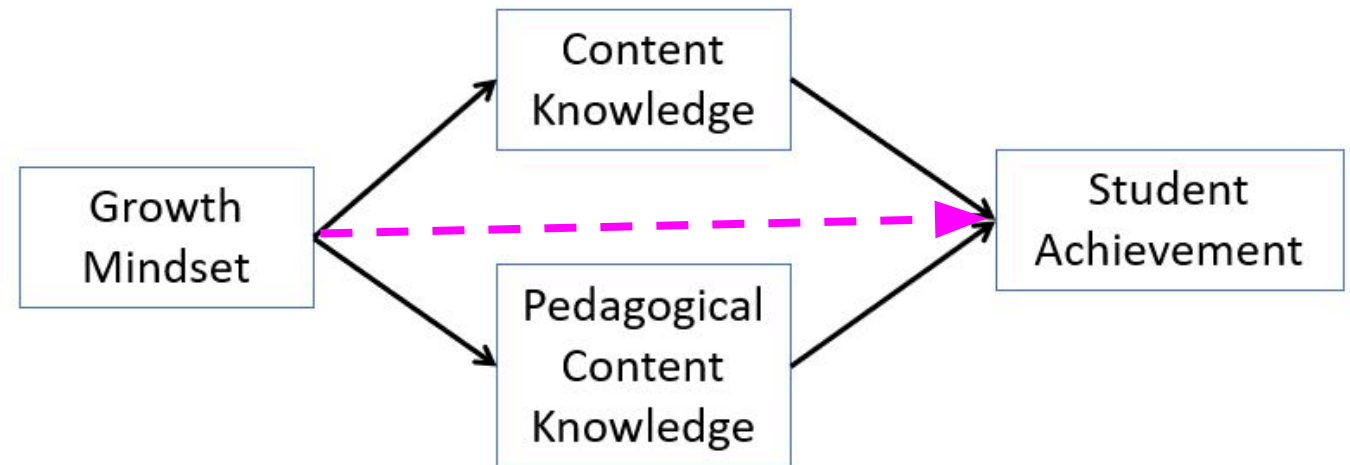
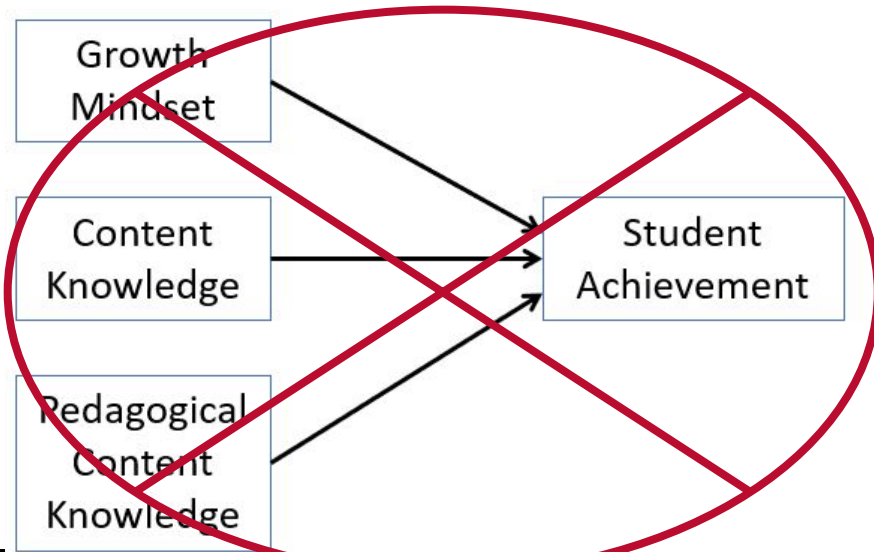
Ph.D. Student in  
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Methodology,  
QMC Research  
Assistant



# QMC Fall Methods Workshop Series

- “Basics of Measurement Modeling”
  - Introduction to *structural equation modeling* (SEM)
  - Slides and materials available at [matthewmadison.com/workshop](https://matthewmadison.com/workshop)
- SEM is comprised of two foundational methods:

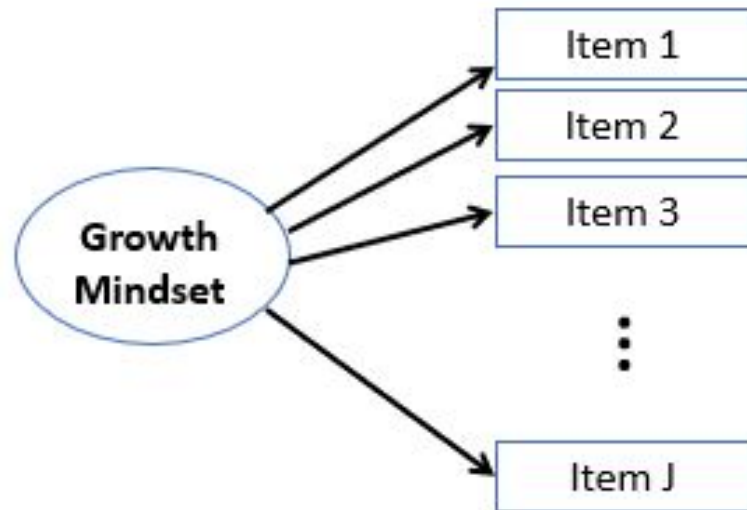
## 1. Path analysis



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- SEM is comprised of two foundational methods:

## 2. Confirmatory Factor Analysis (CFA)

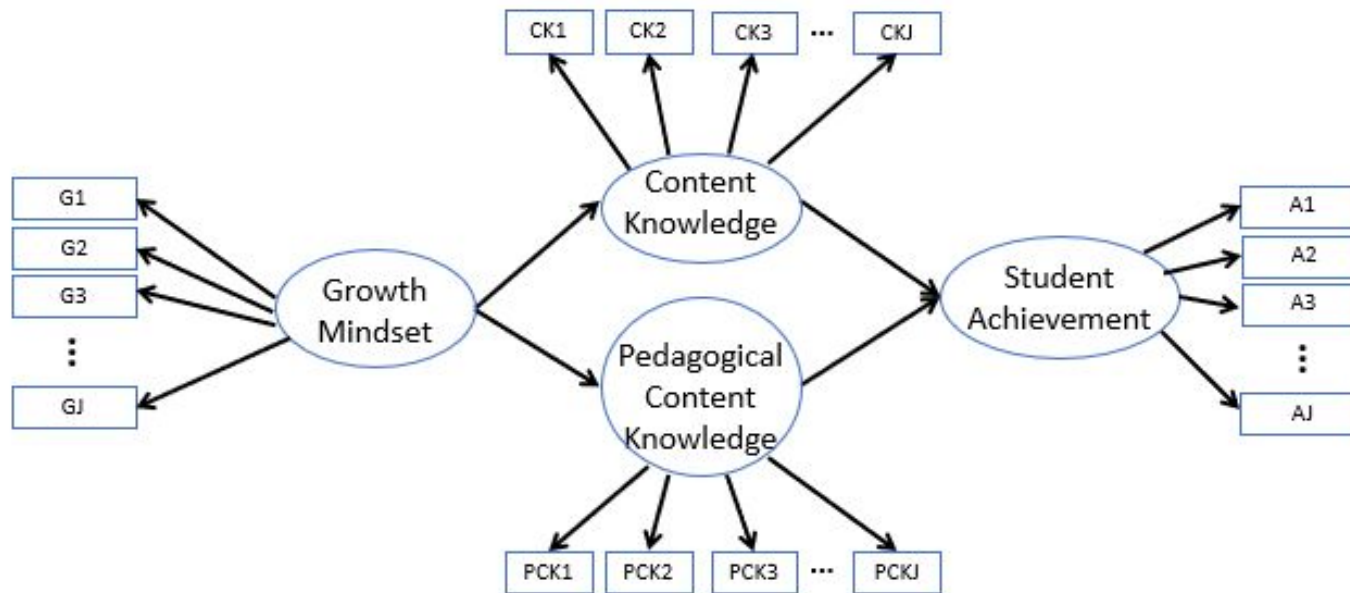


- **Benefits of CFA**
  - Properly model latent variables
  - Assess instrument reliability
  - Assess item quality

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## 3. SEM = path analysis + CFA



- **Benefits of SEM**
  - Model complex relationships, test nuanced theories
  - Handle measurement error appropriately
  - Improved power and inferential accuracy
  - SEM is foundational for many other methods and techniques



# Structural Equation Modeling

A Quantitative Methods Consulting Center Workshop

*Ashley Heady, Quantitative Methodology*



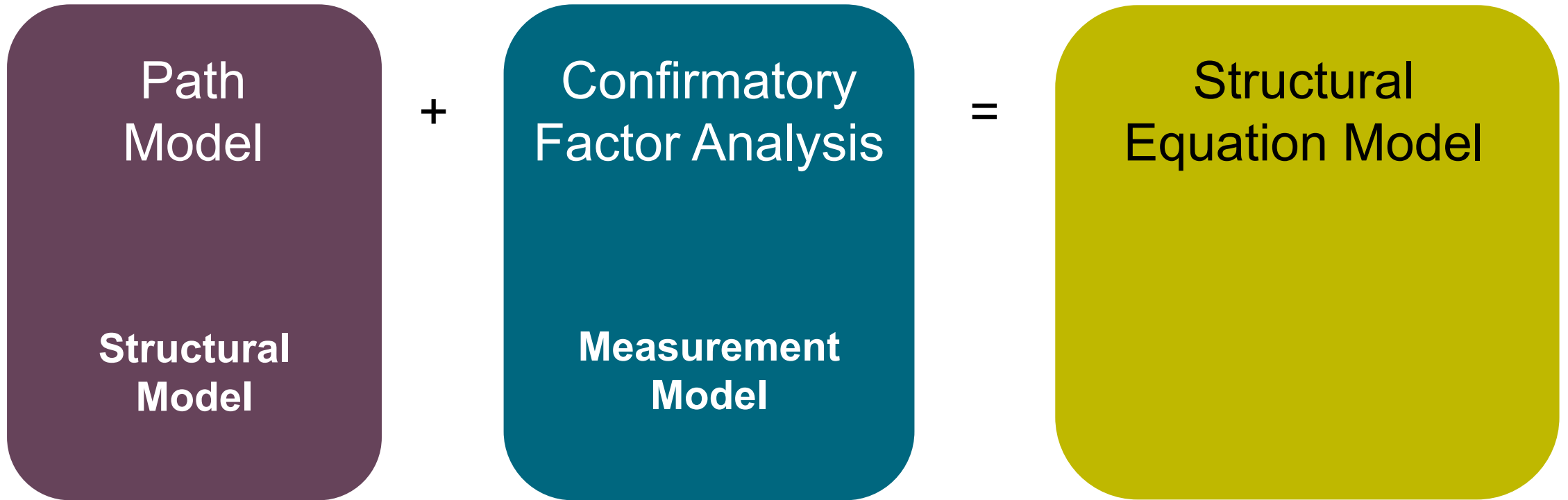
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# Agenda

1. The Structural Equation Model
2. Building an SEM
3. Estimating SEM using R
  - a. Evaluating & tuning model fit
4. Interpretation
5. How to Choose a Model
6. Resources & Extras

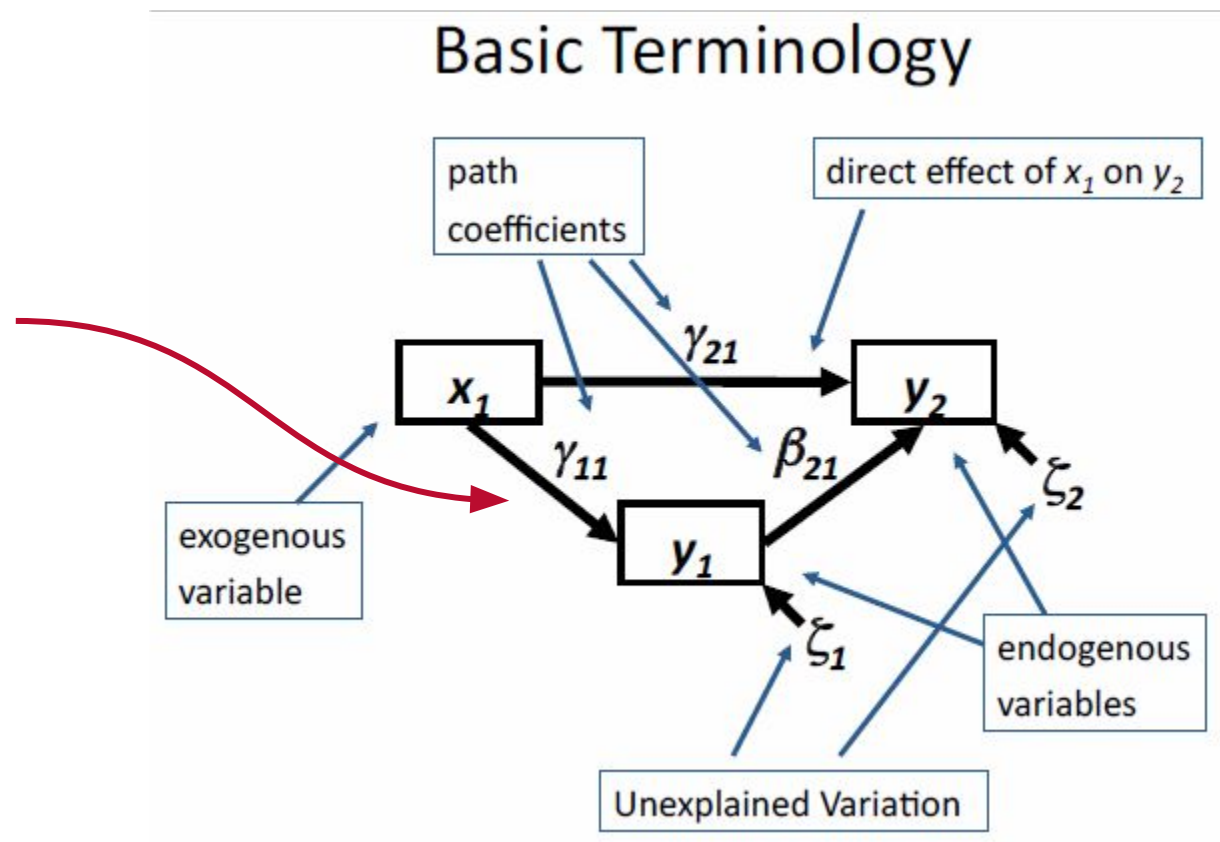


# How it all goes together



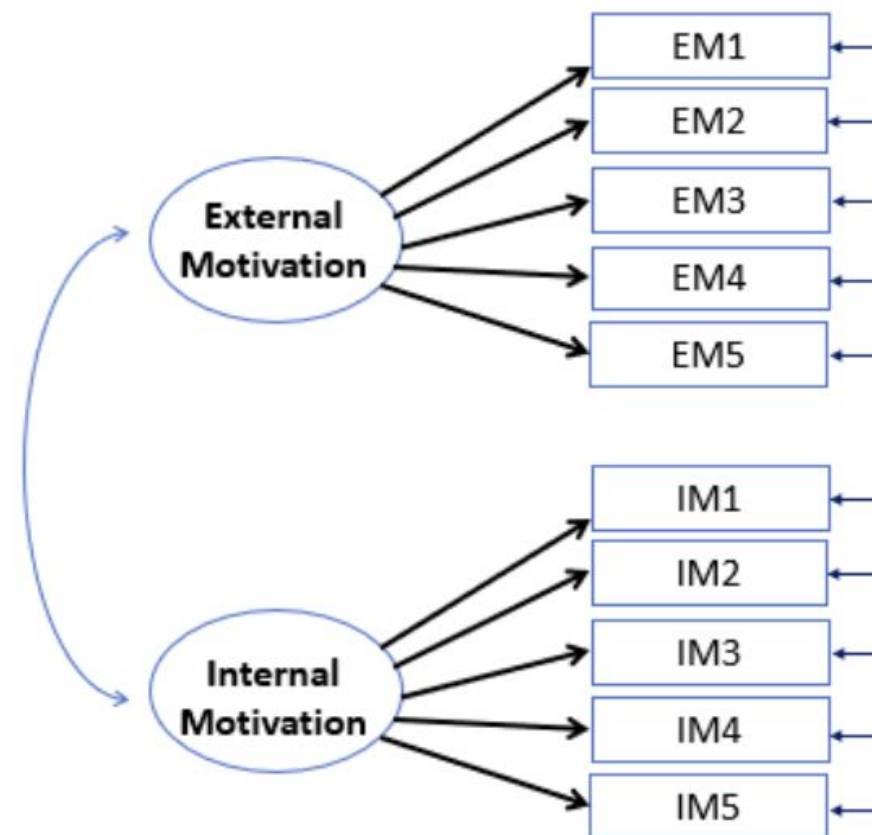
# Refresher of Path Models

- Path models are the structural models in SEM
- They specify a directional relationship or a theorized structure between observed variables



# Refresher of Confirmatory Factor Analysis

- CFA's are the measurement model part of the SEM
- They specify a correlational relationship between observed and latent variables
- Factors should have at least three indicators (for one factor models), or at least two indicators (for models with more than one factor)



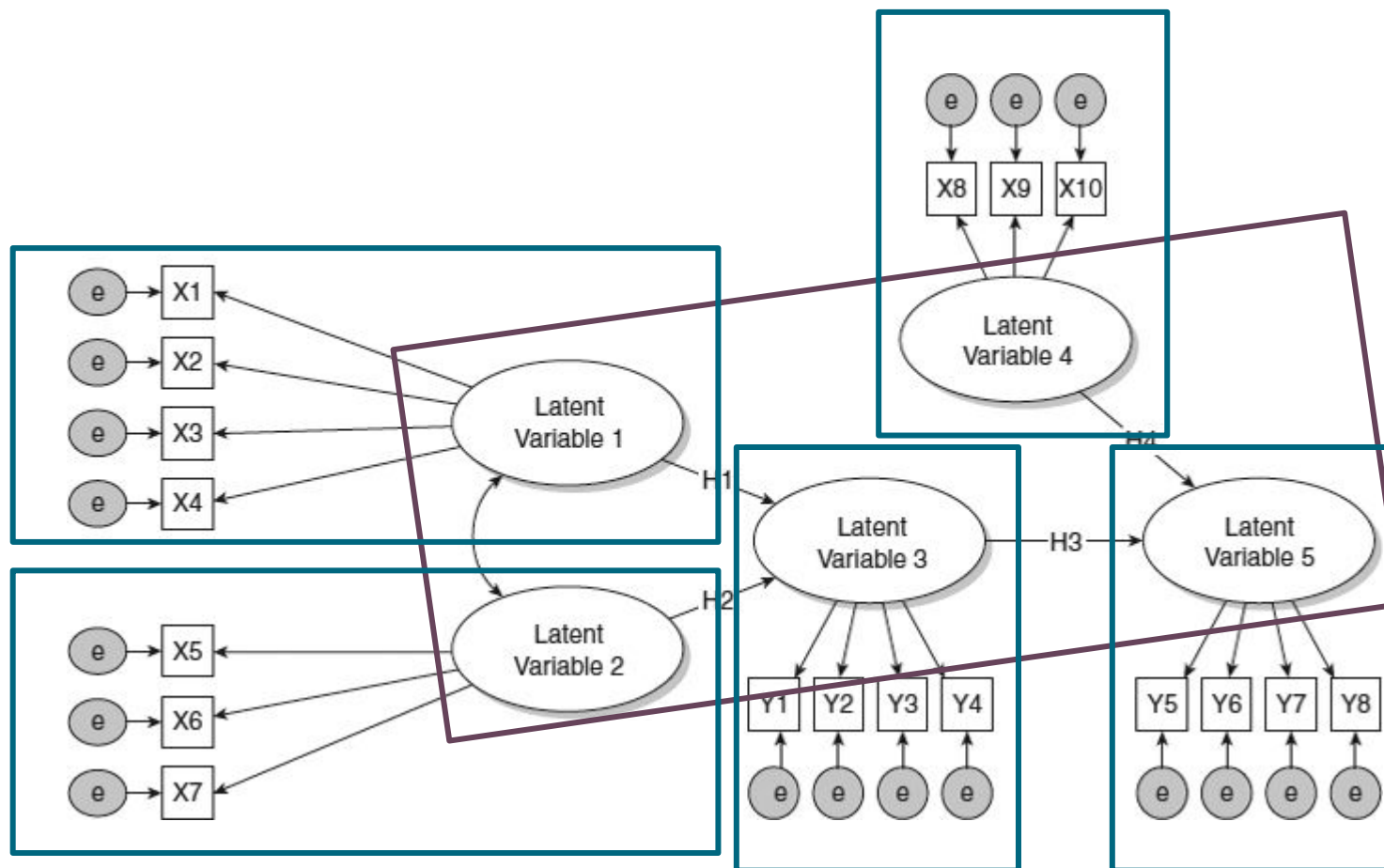
# The Structural Equation Model

Structural Model (path model)

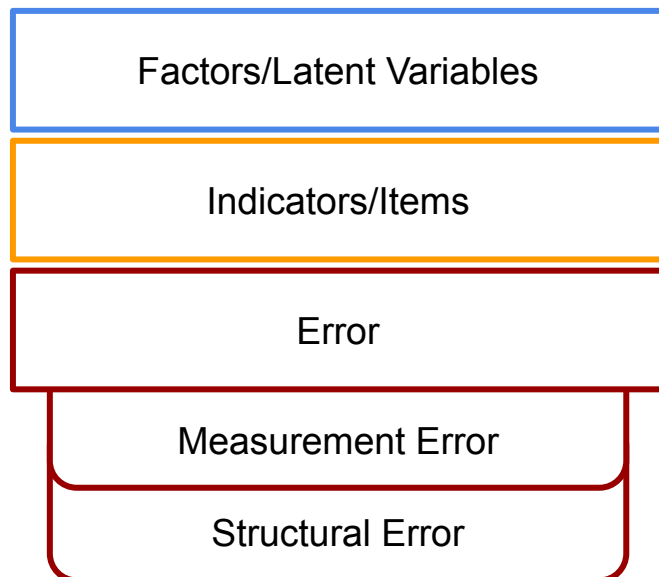
Measurement Model (CFA)

Structural Equation Model

✓ Latent variables now have direct and indirect effects on each other



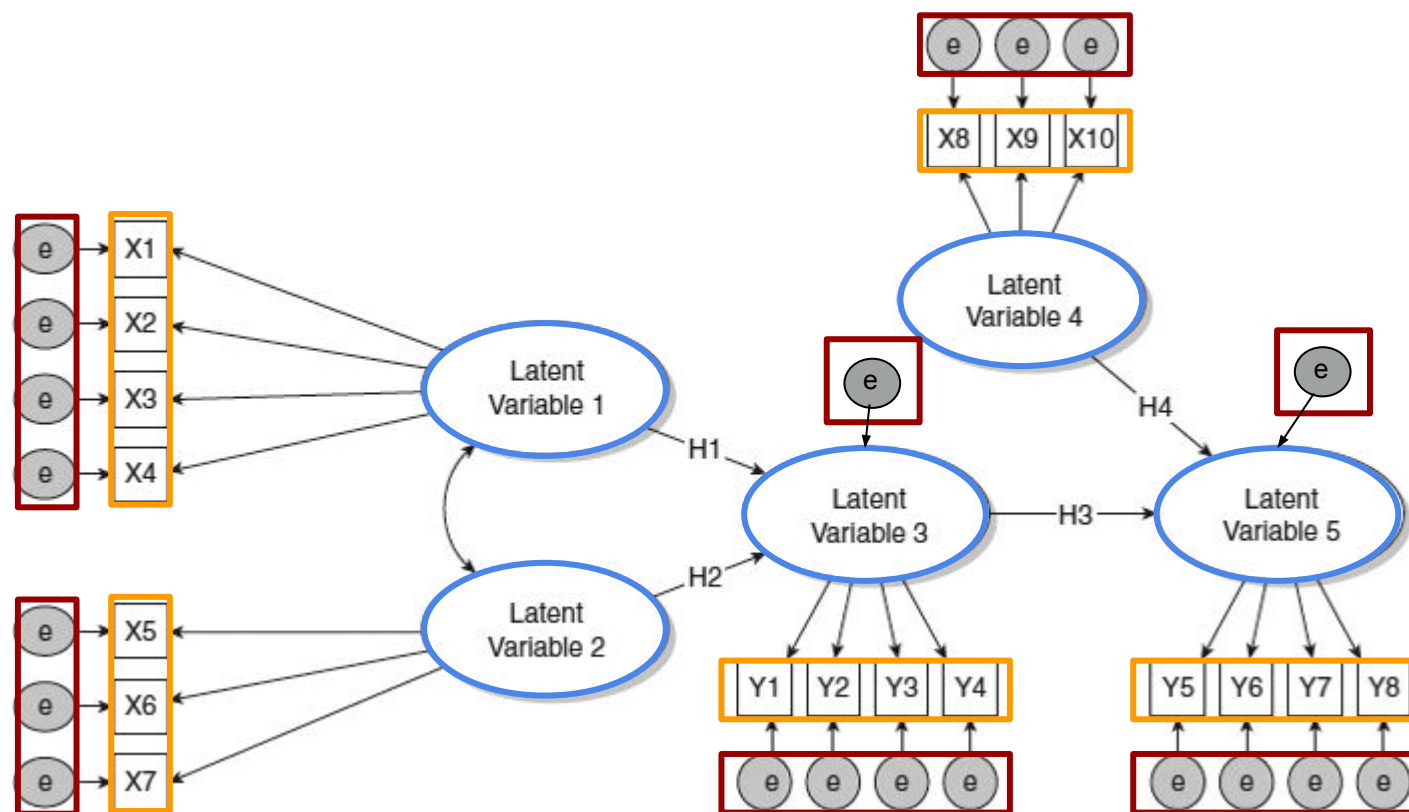
# The Structural Equation Model



Take a look at the shapes!

□ = observed

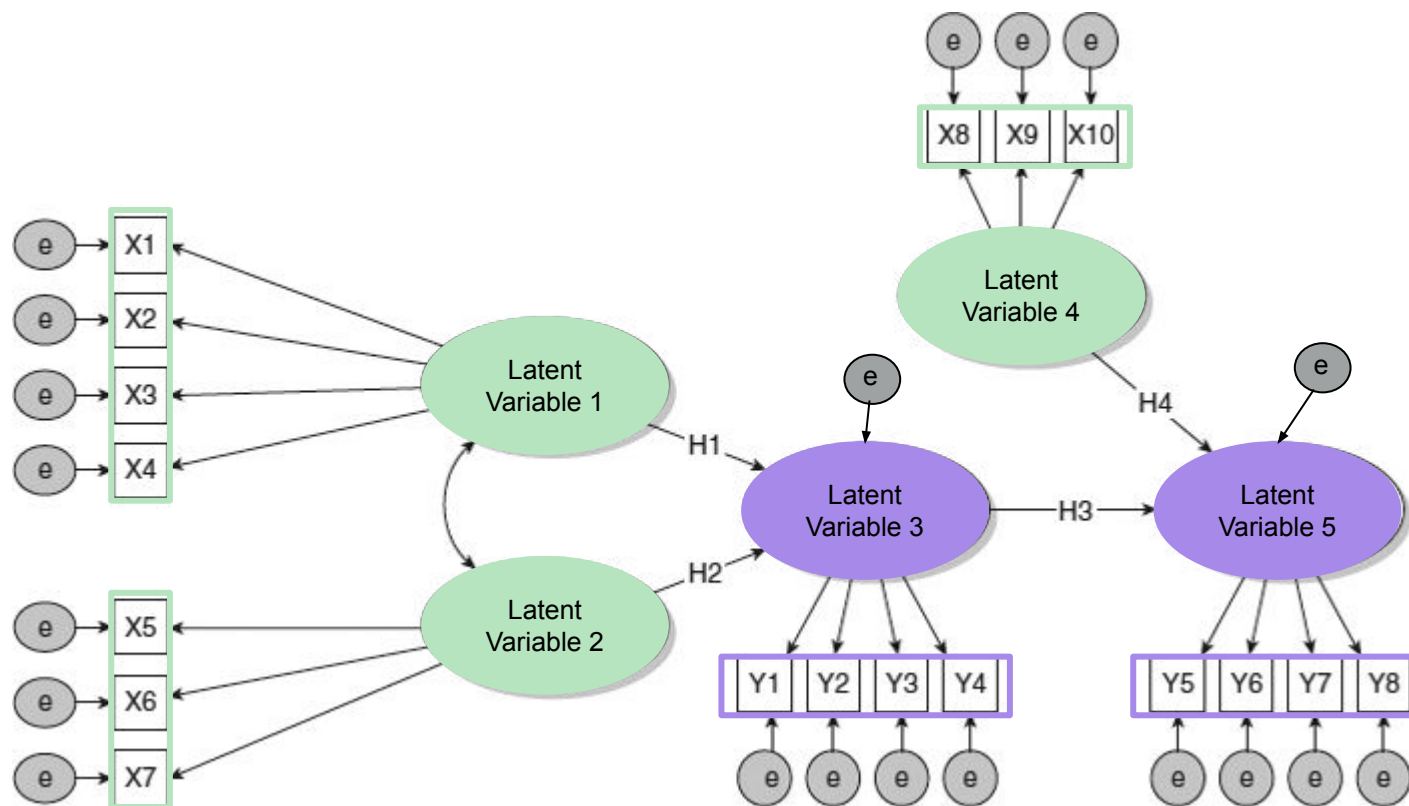
○ = latent



# The Structural Equation Model

Exogenous variables  
(X's, observed & latent)

Endogenous variables  
(Y's, observed & latent)



# Building an SEM

# Building an SEM

1. Get to know your data first
  - a. What do scores in your data set mean?
  - b. What do the distributions look like? (make some histograms!)
2. Extreme differences in variance becomes a problem in SEM
  - a. Simple fix: rescaling (scale() in R)
3. Run a CFA first
  - a. Check that your items are functioning the same across groups (think gender, grade, or ethnicity groups)
4. Identification
  - a. Identification in SEM has to do with the amount of information we have vs the number of parameters we're estimating

Kang, H., & Ahn, J.-W. (2021). Model Setting and Interpretation of Results in Research Using Structural Equation Modeling: A Checklist with Guiding Questions for Reporting. *Asian Nursing Research*, 15(3), 157-162.  
<https://doi.org/10.1016/j.anr.2021.06.001>



# Understanding SEM Identification

Identification starts with the  $t$  rule:  $t \leq \frac{1}{2} (q) (q+1)$

→ basically the number of estimated parameters must be less than or equal to the observed unique entries in the covariance matrix

1

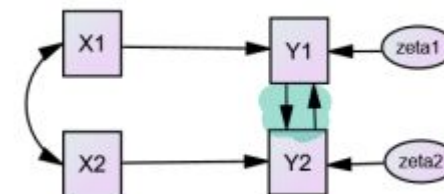
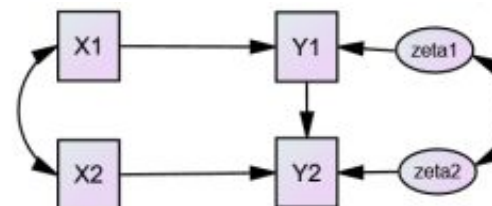
## CFA Identification

- Three rule indicator
  - At least three indicators per latent variable
  - Indicators load to only one latent variable
  - No correlated errors

2

## Structural Identification

- Recursive rules:
  - Are there direct effects with correlated structural error?
  - Do Y's have reciprocal effects?
- If no for both of the above - model is identified



# Understanding SEM Identification

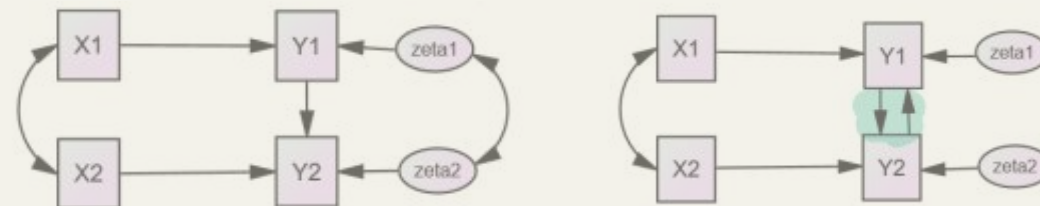
Identification starts with the  $t$  rule:  $t \leq \frac{1}{2} (q) (q+1)$

→ basically the number of estimated parameters must be less than or equal to the observed data

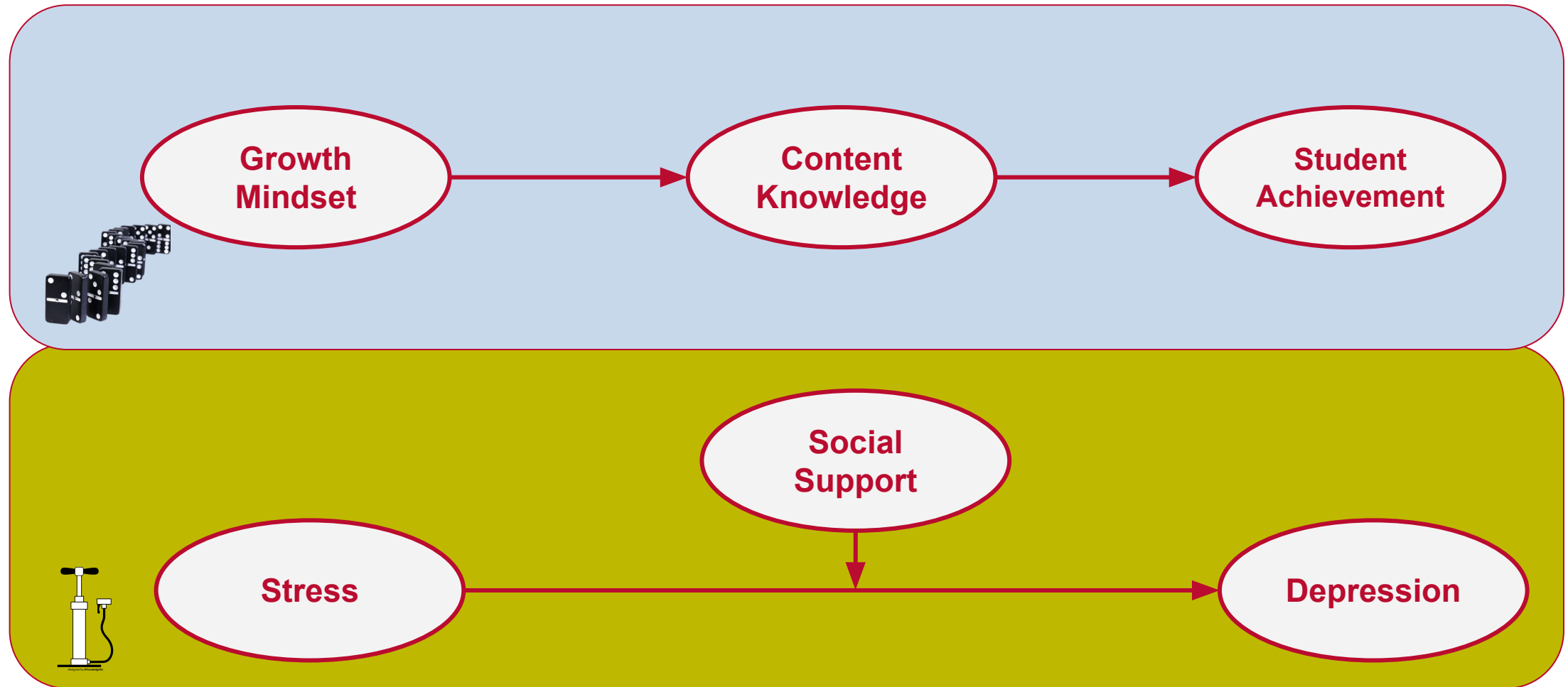
1

## What happens if my model isn't identified?

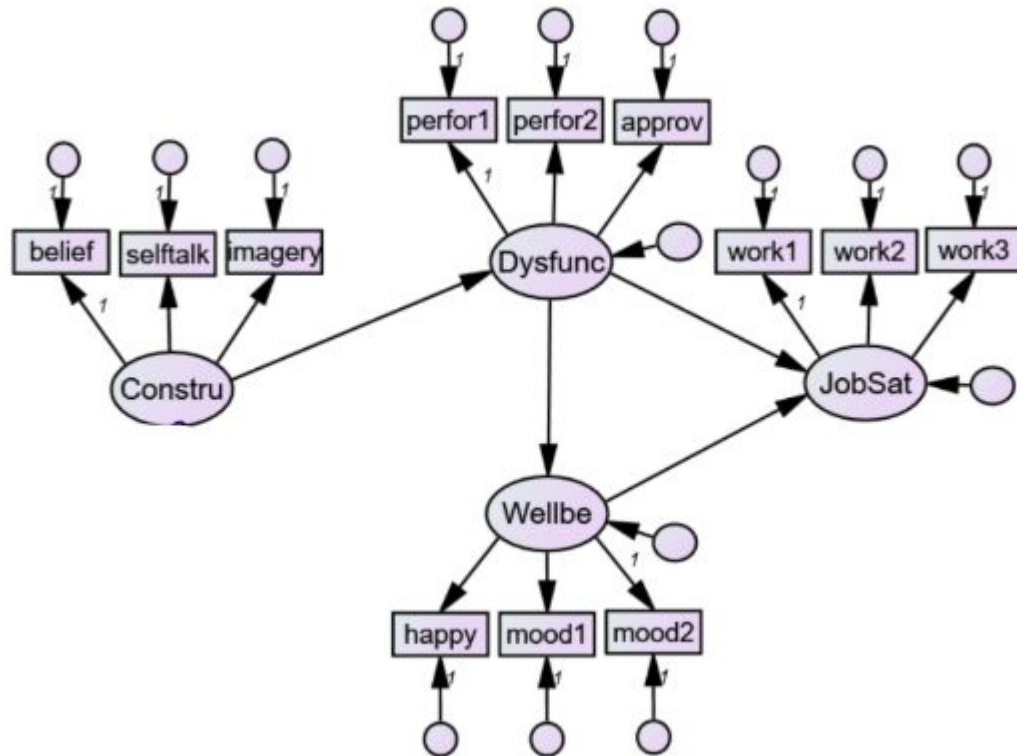
Parameters won't be stable (they'll change every time you run the model) so your output won't be interpretable 😞



# Mediation vs. Moderation



# Example SEM - Direct Relationships Hypotheses



Constru = Constructive thought strategies

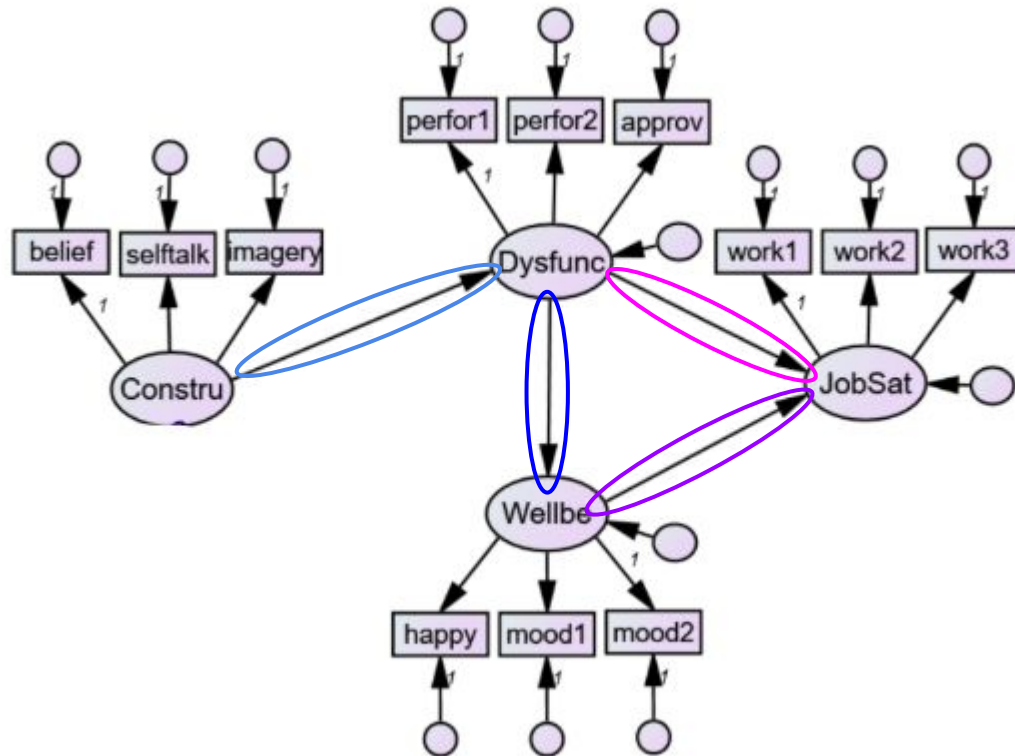
Dysfunc = dysfunctional thought process

Wellbe = Subjective well-being

JobSat = Job satisfaction

Houghton, J.D., Jinkerson, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 45-53 (2007).  
<https://doi.org/10.1007/s10869-007-9046-9>

# Example SEM - Direct Relationships Hypotheses



## Hypothesis 1

Subjective well-being will be positively related to job satisfaction.

## Hypothesis 2

Dysfunctional thought processes will be negatively related to subjective well-being.

## Hypothesis 3

Constructive thought strategies will be negatively related to dysfunctional thought processes; . . .

## Hypothesis 4

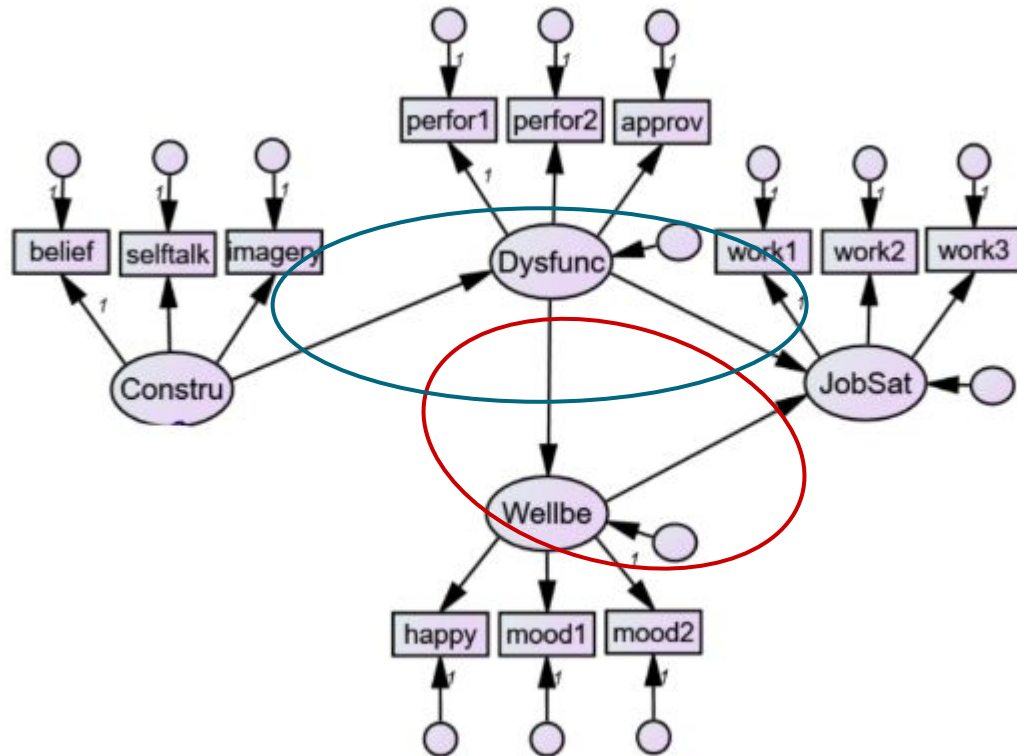
Dysfunctional thought processes will be negatively related to job satisfaction.

## How do we test these hypotheses?

→ Check the significance of the regression coefficients

Houghton, J.D., Jinkerson, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 45-53 (2007). <https://doi.org/10.1007/s10869-007-9046-9>

# Example SEM - Indirect/Mediating Relationships



## Hypothesis 5

The effects of dysfunctional thought processes on job satisfaction will be partially mediated through subjective well-being.

## Hypothesis 6

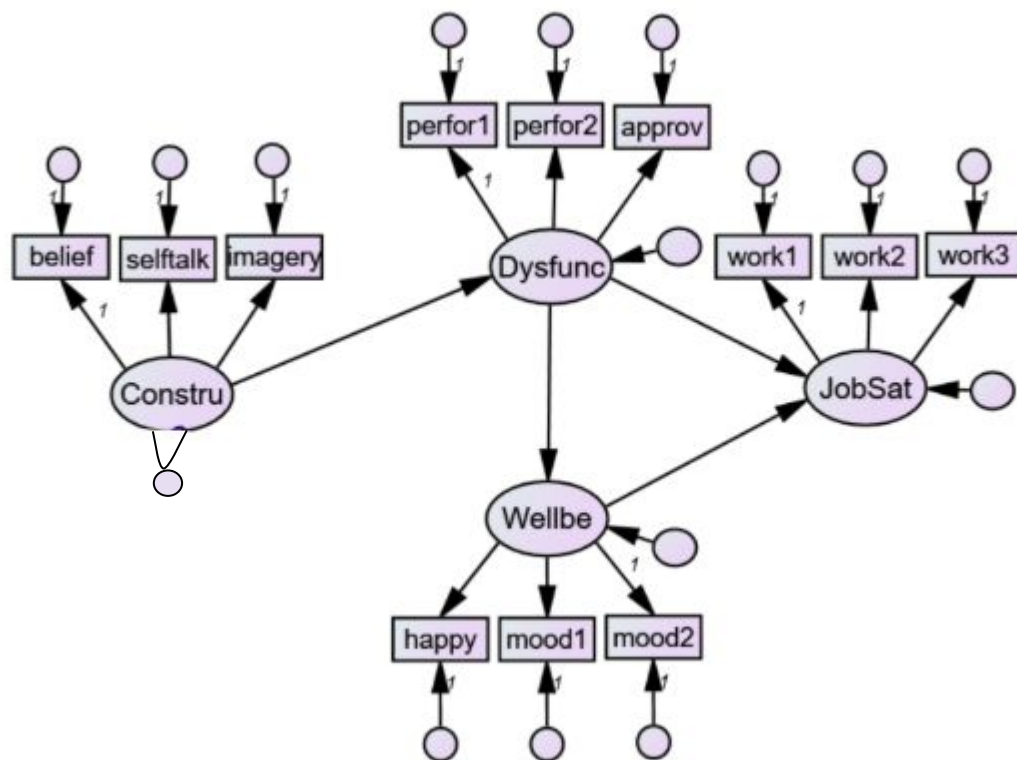
The effects of constructive thought strategies on job satisfaction will be fully mediated through dysfunctional thought processes and subjective well-being.

## How do we test these hypotheses?

→ Compare models with & without mediation

Houghton, J.D., Jinkerson, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 45-53 (2007).  
<https://doi.org/10.1007/s10869-007-9046-9>

# Example SEM - Identification



## t-Rule

✓ 28 parameters  $\leq$  78 unique entries in covariance matrix  $((12 \times 13)/2)$

## Three-indicator rule

✓ Three indicators per latent variable  
✓ Indicators load to only one latent variable  
✓ No correlated errors

## Structural Identification

✓ No recursive relationships  
✓ No correlated structural error

# Estimating SEM in R

# Estimating Path Models in R



**Install lavaan & open the library**



**Define the covariance matrix & relationships**



**Run the model**



**Evaluate the fit**

# The Code

```
install.packages(lavaan); library(lavaan)
```

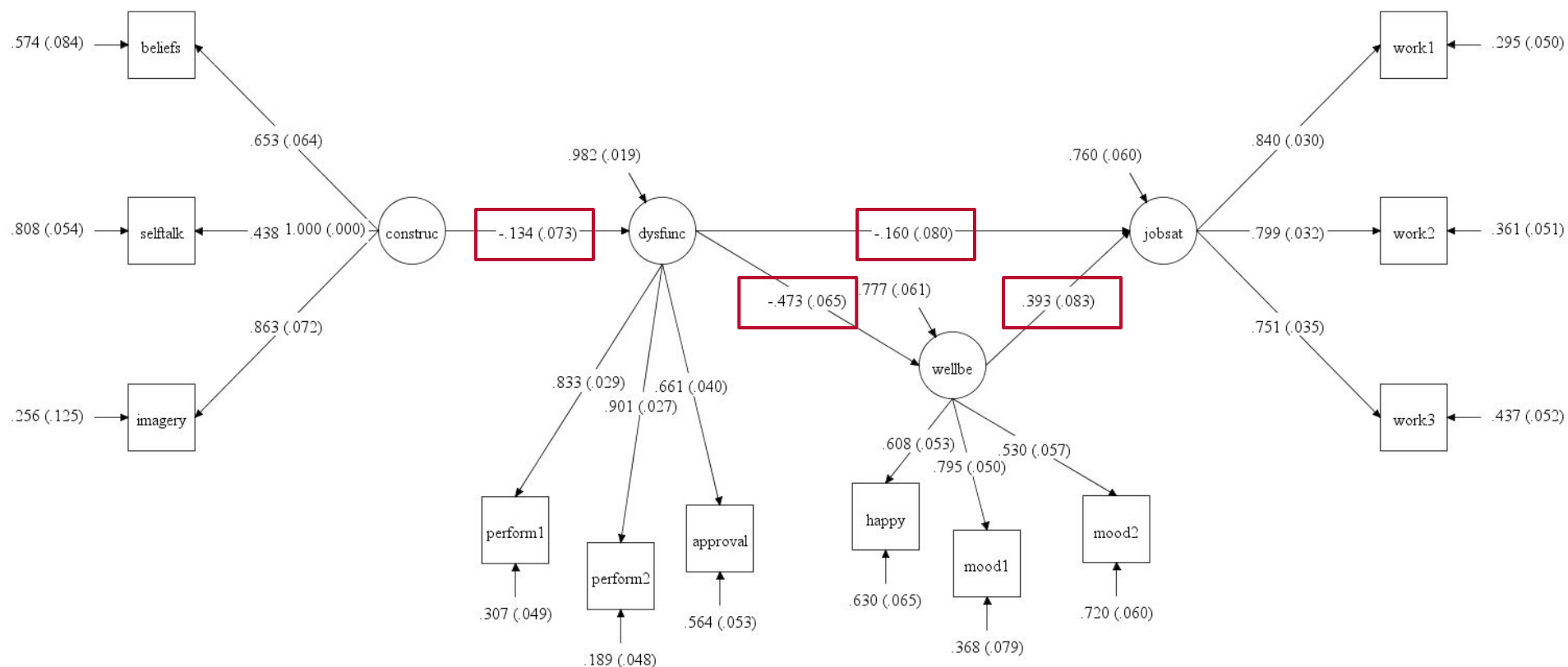
```
sem_model <- '  
construc =~ beliefs + selftalk + imagery  
dysfunc =~ perform1 + perform2 + approval  
wellbe =~ happy + mood1 + mood2  
jobsat =~ work1 + work2 + work3  
dysfunc ~ construc  
wellbe ~ dysfunc  
jobsat ~ dysfunc + wellbe'
```

```
fit <- sem(sem_model, sample.cov=cov, sample.nobs=263)
```

```
summary(fit, fit.measures=TRUE, standardized=TRUE)  
resid(fit, type='standardized'); modindices(fit)
```



# Example SEM - with Estimates



Houghton, J.D., Jinkerson, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. J Bus Psychol 22, 45-53 (2007). <https://doi.org/10.1007/s10869-007-9046-9>

# Model Fit Evaluation

## Chi-Squared

Null hypothesis  
is essentially  
“the model fits”

### Looking for:

- $p\text{-value} > 0.05$

## RMSEA

Two outputs:  
> RMSEA estimate  
> RMSEA Confidence  
Interval

### Looking for:

- $\text{RMSEA} \leq 0.05$

## CFI/TLI

CFI & TLI compare your  
proposed model to the  
baseline model

### Looking for:

- $\text{CFI \& TLI} \geq 0.90$



# Model Tuning

## Residuals

> Shows you the difference between your actual covariance matrix and the fitted model covariance

## Modification Indices

> Shows you paths and covariances that could be estimated in the model that would make the model fit better

### Looking for:

- Values  $> 2$  are problematic

### Looking for:

- Values  $> 4 \rightarrow$  let the model estimate



# Things to remember in SEM

- Draw out your model first! How does it look?
- Complex models are hard to interpret & require lots of data! Keep it simple
- Remember to look at all the fit statistics, they come as a package (but remember chi-squared is very sensitive!)
- Your proposed model does not prove causality, it's just an assumption of how things work



# How to Choose a Model

# Choosing a Model

Least Complex

Most Complex

|             | Multiple Regression       | Path Model              | CFA                    | SEM                          |
|-------------|---------------------------|-------------------------|------------------------|------------------------------|
| Goal        | Description or Prediction | Observational Structure | Instrument Reliability | Complex Latent Relationships |
| Sample Size | ~30-50+                   | ~60-100+                | ~90-400+**             | ~90-400+**                   |
| Inferences  | Observational             | Observational           | Latent                 | Latent                       |

\*\*Wolf, E. J., Harrington, K. M., Clark, S. L., & Miller, M. W. (2013). Sample Size Requirements for Structural Equation Models: An Evaluation of Power, Bias, and Solution Propriety. *Educational and psychological measurement*, 76(6), 913-934. <https://doi.org/10.1177/0013164413495237>



# A note on CFA & SEM Sample Size

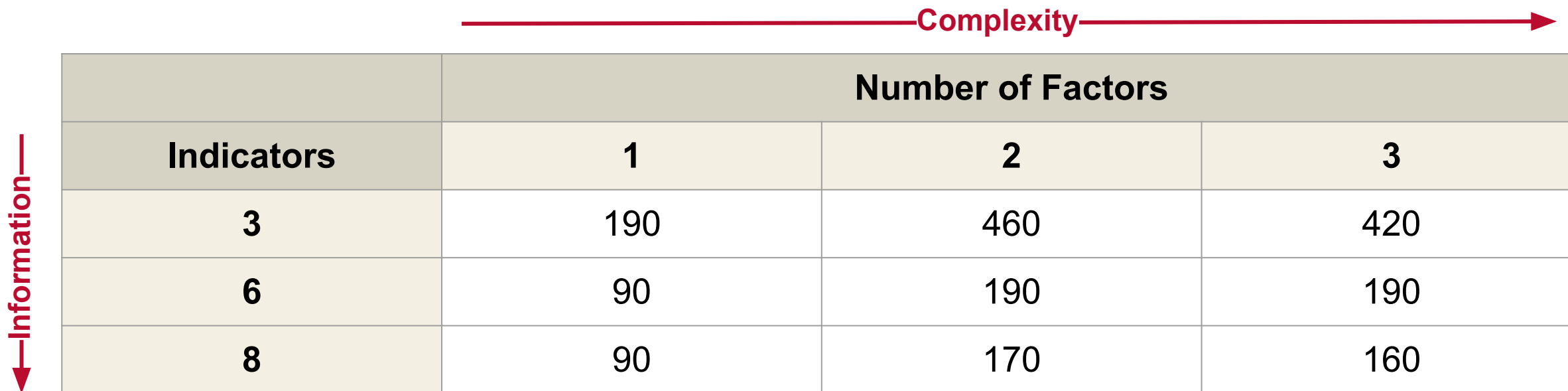
Proper sample size is impacted by:

- Number of factors
- Indicators per factor
- Missingness
- Strength of relationships

\*\*Wolf, E. J., Harrington, K. M., Clark, S. L., & Miller, M. W. (2013). Sample Size Requirements for Structural Equation Models: An Evaluation of Power, Bias, and Solution Propriety. *Educational and psychological measurement*, 76(6), 913-934.  
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# Sample Size\*



| Indicators | Number of Factors |     |     |
|------------|-------------------|-----|-----|
|            | 1                 | 2   | 3   |
| 3          | 190               | 460 | 420 |
| 6          | 90                | 190 | 190 |
| 8          | 90                | 170 | 160 |

\*Assuming no missing data and moderate relationships (loadings = .50)

\*\*Wolf, E. J., Harrington, K. M., Clark, S. L., & Miller, M. W. (2013). Sample Size Requirements for Structural Equation Models: An Evaluation of Power, Bias, and Solution Propriety. *Educational and psychological measurement*, 76(6), 913-934.  
<https://doi.org/10.1177/0013164413495237>



# Resources & Extras

# Resources for Path Models & SEM

## Lavaan Website

[lavaan.ugent.be/  
tutorials & resources](http://lavaan.ugent.be/tutorials)

## YouTube

CenterStat, QuantFish,  
our QMC YouTube channel

### Courses:

- **ERSH 7600**: construction of measurement instruments
- **ERSH 8130**: survey of latent variable methods
- **ERSH 8750**: structural equation modeling
- **ERSH 8740**: exploratory factor analysis

### Books:

- **Kline**: Principles and practice of structural equation modeling
- **Bollen**: Structural equations with latent variables
- **Loehlin**: Latent variable models: An introduction to factor, path, and structural equation analysis



# More Questions?

Visit us at the Quantitative  
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***Thanks for coming!***